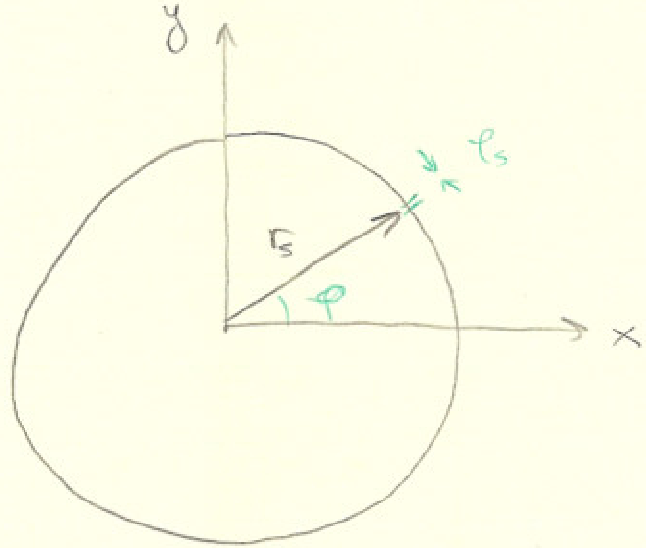
Magnetic feild of a current loop....



Find $\vec{H}$ @ $(0, 0, z_p)$

For the peice of wire @ $(x_s, y_s, 0)$

$$= (r\cos\varphi_s, r\sin\varphi_s, 0)$$

$$d\vec{l}_s = (-r\sin\varphi_s\, d\varphi_s, r\cos\varphi_s\, d\varphi_s, 0)$$

$$\vec{r_p} - \vec{r_s} = -r\cos\varphi_s\,\hat{x} - r\sin\varphi_s\,\hat{y} + z_p\hat{z}$$

$$d\vec{l} \times (\vec{r_p} - \vec{r_s}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -r\sin\varphi_s d\varphi_s & r\cos\varphi_s d\varphi_s & 0 \\ -r\cos\varphi_s & -r\sin\varphi_s & z_p \end{vmatrix}$$

$$= \hat{x}(z_p r\cos\varphi_s d\varphi_s) + \hat{y}(r z_p \sin\varphi_s d\varphi_s) + \hat{z}(r^2 d\varphi_s)$$

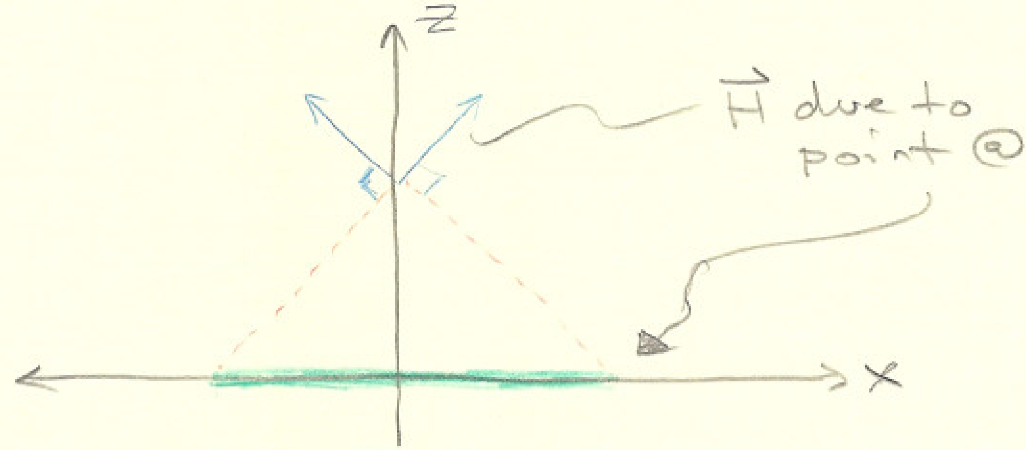
$$H_x = \int_0^{2\pi} \frac{1}{4\pi} I \frac{z_p r \cos\varphi_s d\varphi_s}{(r^2 + z_p^2)^{3/2}} = \frac{1}{4\pi} \frac{I z_p r}{(r^2+z_p^2)^{3/2}} \int_0^{2\pi} \cos\varphi_s d\varphi_s$$

$$= \emptyset$$

$$H_y = \int_0^{2\pi} \frac{1}{4\pi} I \frac{z_p r \sin\varphi_s d\varphi_s}{(r^2+z_p^2)^{3/2}} = \emptyset$$

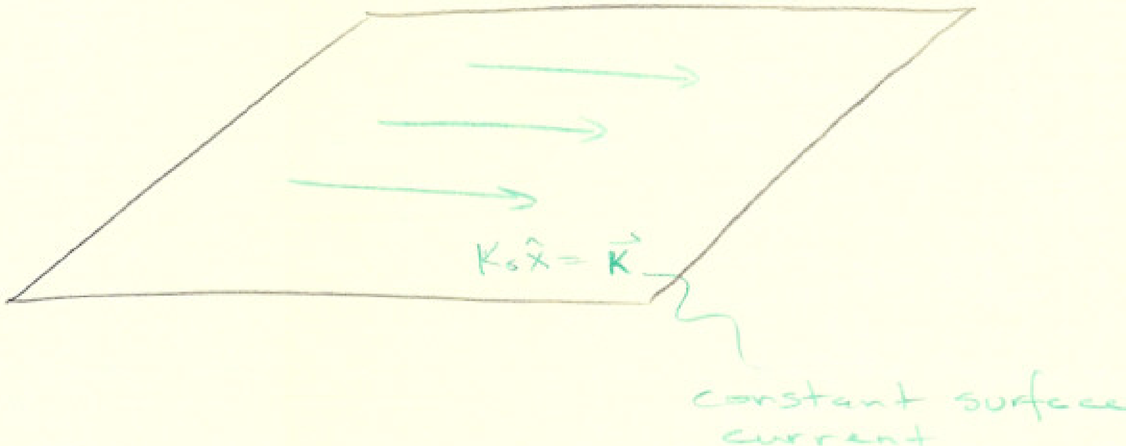
$$H_z = \int_0^{2\pi} \frac{1}{4\pi} I \frac{r^2 \, d\varphi_s}{(r^2+z_p^2)^{3/2}} = \frac{I}{2} \frac{r^2}{(r^2+z_p^2)^{3/2}}$$

$$\vec{H} = \frac{I}{2} \frac{r^2}{(r^2+z_p^2)^{3/2}}$$ compare to $\vec{E}$



Loop is the prototype of the magnetic dipole.

Infinite Sheet



An infinite sheet @ $z=0$ carries constant $\vec{K} = K_0\hat{x}$

What is $\vec{H}$ @ $\vec{r}_p = \langle 0, 0, z_p \rangle$?

What is $d\vec{S}$? (break the sheet into little surfaces)

$$\vec{r}_s = \langle x_s, y_s, 0 \rangle$$

$$ds = dx_s\, dy_s$$

$$\vec{K}\, ds = K_0 \hat{x}\, dx_s\, dy_s$$

$$\vec{r}_p - \vec{r}_s = -x_s\hat{x} - y_s\hat{y} + z_p\hat{z}$$

$$\vec{K}\, ds \times (\vec{r}_p - \vec{r}_s) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ K_0 dx_s dy_s & 0 & 0 \\ -x_s & -y_s & z_p \end{vmatrix}$$

$$= -\hat{y} K_0 z_p\, dx_s\, dy_s - \hat{z} K_0 y_s\, dx_s\, dy_s$$

$$d\vec{H} = \frac{K_0}{4\pi} \frac{(-\hat{y} z_p dx_s dy_s - \hat{z} y_s \cancel{K_0}\, dx_s dy_s)}{(x_s^2 + y_s^2 + z_p^2)^{3/2}}$$

$$H_y = -\frac{K_0 z_p}{4\pi} \int_{-\infty}^{\infty} dx_s \int_{-\infty}^{\infty} dy_s \frac{1}{(x_s^2 + y_s^2 + z_p^2)^{3/2}}$$

Let $x_s = r_s \cos \varphi_s$

$y_s = r_s \sin \varphi_s$

$r_s = \sqrt{x_s + y_s}$

$\tan \varphi_s = y_s / x_s$

$dx_s dy_s \rightarrow d\varphi_s\, r_s dr_s$

$$H_y = -\frac{K_0 z_p}{4\pi} \int_0^{\infty} r_s dr_s \int_0^{2\pi} d\varphi_s \frac{1}{(r_p^2 + z_p^2)^{3/2}}$$

$$= -\frac{K_0 z_p}{2} \int_0^{\infty} dr_s \frac{r_s}{(r_s^2 + z_p^2)^{3/2}}$$

$$= -\frac{K_0 z_p}{2} \frac{1}{|z_p|}$$

$$H_z = -\frac{K_0}{4\pi} \int_{-\infty}^{\infty} dx_s \int_{-\infty}^{\infty} dy_s \frac{y_s}{(x_s^2 + y_s^2 + z_p^2)^{3/2}}$$

$$= \frac{K_0}{4\pi} \int_0^{\infty} r_s dr_s \int_0^{2\pi} d\varphi_s \frac{r_s \sin \varphi_s}{(r_s^2 + z_p^2)^{3/2}} = 0$$

$$\vec{H} = -\hat{y}\,\frac{K_0}{2}\begin{cases} +1 & \text{if } z_p > 0 \quad z_p/|z_p| = 1 \\ -1 & \text{if } z_p < 0 \quad z_p/|z_p| = -1 \end{cases}$$

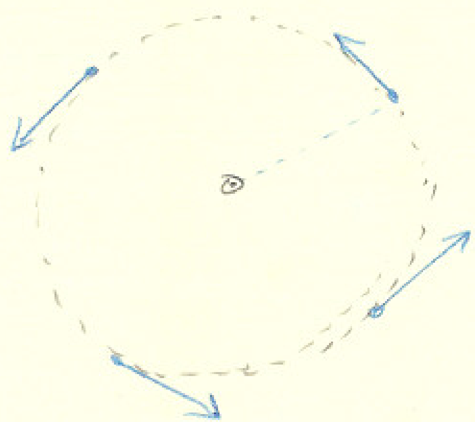
Recall $\vec{E} = \frac{\sigma_0}{2\varepsilon_0} \pm \hat{z}$:) quite a comparison

Ampere's Law

Ampere's Law is similiar to Gauss's Law but w.r.t. $\vec{H}$ instead of $\vec{E}$.

Feild of an infinitely straight line.

$$\boxed{\int_c \vec{H}\cdot d\vec{l} = I_{enclosed}}$$



$d\vec{l} = r\,d\varphi\,\hat{\varphi}$

$$\oint \vec{H}\cdot d\vec{l} = 2\pi r\,|\vec{H}(r)| = I_{enclosed}$$

$$\therefore \quad \vec{H}(r) = \frac{I_{enclosed}}{2\pi r}\,\hat{\varphi}$$